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Seventh Semiannual Status Report covering the period 1 February 1981 - 31 July 1981

(NASA-CR-164939) RESEARCH DEVELOPING CLOSED N82-12084
LOOP BOLL CONTROL FOR MAGNETIC BALANCE
SYSTEMS Semiannual Status Report, 1 Feb. 31 Jul. 1981 (Massachusetts Inst. of Tech.) Unclas
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Research Developing Closed Loop Roll Control for Magnetic Balance Systems

> NASA, Langley Research Center (Grant No. NSG 1502) OSP 86251

Professor Eugene E. Covert and Dr. Charles W. Haldeman, Principal Investigators



Work during the period

During the past six months effort was directed to the following areas:

- Procurement and installation of the line printer,
 A-D converter boards, clock and extra memory for the MINC 11-23 minicomputer, and the logic analyzer.
- 2. Interconnection of the computer inputs to magnetic balance outputs to provide computer position control and data acquisition. This necessitated construction of four junction boxes and two 27 pair cables 70 feet long to provide connections from balance to computer and computer to balance.
- 3. Completion of the investigation of the use of parameter identification of a means of determining dynamic characteristics. This study indicated that the technique was successful at providing preliminary measurements of $C_{m_{\dot{\alpha}}} + C_{m_{\dot{q}}}$ for the ogive cylinder model. Sufficient measurements were not made to provide a comparison with other methods; i.e., forced oscillation motion. The measurements made indicated that results could be obtained with random noise inputs of .1, .2 and .4 degrees in pitch. This is reported in Gautham Ramohalli's M.S. thesis, an abbreviated version of which constitutes the bulk of this report.
- 4. Repair of Thyratron and motor generator power supplies for the pitch and yaw degrees of freedom. Vacuum tubes and defective wiring in the Thyratron power supply used in pitch were

Stability Derivative Estimation using Digital Signal Processing Methods with the Magnetic Suspension Wind Tunnel, S.M. Thesis, Dept of Aeronautics and Astronautics, MIT, May, 1981.

replaced to permit use of the supply during extensive repair of the motor generators. Time since overhaul on these supplies was about 1100 hours, which is 100 hours longer than the expected life of the generator bearings.

Because of the increased effort required for computer connection and system repairs, grant funds have been expended as of this reporting period.

Personnel

During the past six months the following professional personnel have contributed to the effort under this program.

Professor Eugene E. Covert

Dr. Charles W. Haldeman

Mr. Gautham Ramohalli

Choice of a method for handling dynamic system data

The first method considered was the EEIC method of Merhav and Gabay (1). This has the attractive feature of being able to estimate not only poles but zeroes of a continuous system directly. Further, the method drives the pole zero excess coefficients (if a higher order model is guessed) automatically to zero and any excess pole zero pairs are driven to each other.

This method was tried on a simple first order model on a simulation on the VAX 11/780. The method showed good convergence to reasonably accurate values. However, it needed about 50 seconds of data before the values settled down. As the order of the model is increased, the complexity of the

process increases. For each order increase an additional filter is required of the form s/(s+a)^k. Further, the pole zero pairs need to be monitored and factorization used to cancel any poles and zeroes that converge to each other (but do not converge to a steady value!). This method was not used.

If the memory and computational requirements are to be minimized, analog filters are indicated. However, since evaluating digital data processing for the application is an important goal, the data was processed using the MANC.

Since the MINC was used to acquire many data points in a short time, it was located near the wind tunnel. To permit its use for data reduction as well as acquisition, a reduction method was selected that could be easily implemented on the MINC rather than having to transfer the data over to the larger central VAX system. This led to choice of the second method.

The second method considered was that of the Fourier spectrum analysis using Fast Fourier Transforms algorithms.

Using the Institute of Electrical and Electronics Engineers digital signal processing program (2), FFT subroutines were implemented on the MINC 11/23 described below. However, spectrum analysis methods have the shortcoming that their resolution in frequency is limited by the length of data 'T' as

 $\Delta f \ge \frac{1}{T}$ (uncertainty principle)

This is similar to a problem encountered in processing speech signals where the length of the data stream is limited due to the time varying nature of the vocal signals. In fact, it parallels the problem closely if we consider large amplitude motion where the dynamics are essentially non-linear and a time varying small increment linear model for the airplane and wind is needed.

The method used there is to model the signal using co-variance/correlation merbods digitally. This is based on the least squares norm minimization and is referred to as MEM or maximum entropy method.

The method relies on statistical covariance/correlation approach to estimating the poles of a <u>digital</u> filter using short streams of data and has the desirable property that it eliminates the restriction imposed by the uncertainty principle by effectively "making more data" using an analytical model of the system. Of course, this method <u>has</u> the obvious drawback that if the generated model is erroneous, then the data generated using it is meaningless. However, the method <u>has</u> been successfully used in the speech case and is presently being applied to speech recognition.

Further, the computational and memory requirements are quite modest and an estimate of the accuracy of the model is

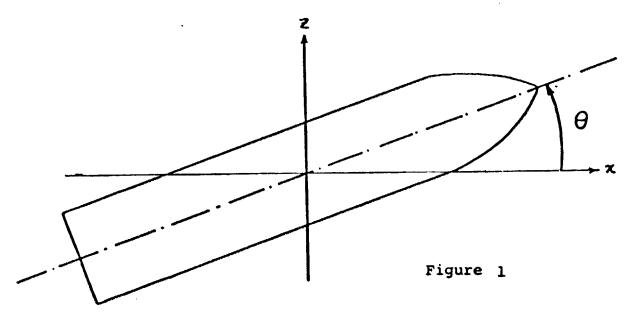
based on the residual energy. It can be used to decide when the model order is high enough if a suitable criteria on the size of the residual is available. This was the method chosen for use.

Application to the magnetic balance

The procedure followed was to adjust the model suspension to be very stiff in all degrees of freedom except pitch. The pitch position response was then stored during a run and analyzed using the MINC. For this case all displacements except pitch are negligible and the equation of motion for a second order system with a natural frequency ω_n and a damping ratio ζ is

$$\frac{d^2\theta}{dt^2} + 2\zeta\omega_n \frac{d\theta}{dt} + \omega_n^2\theta = M \dots$$

where M is the forcing moment and θ is the angle of the body axis from the tunnel axis. See Figure 1.



Equation 1 in a more transparent form is

$$J \frac{d^2\theta}{dt^2} + D_{M_p} \frac{d\theta}{dt} + K_{M_p} \theta = P(t) + M_A(t)$$
 la

M_A = Aerodynamic moment function

 $D_{M_{p}}$ = Magnetic damping

J = Moment of inertia about centre of rotation

 $K_{M_{D}}$ = Stiffness due to magnetic suspension system

P(t) = Pitch input driving function

The aerodynamic moment can be linearized as follows. An M_{Θ} term that is due to the lift force due to an angle of attack. An M_{Θ}^* term that is due to rotation of any point along the body and an $M_{\tilde{G}}$ term that comes about due to motion of points along the body (except at centre of rotation) in the z direction. The total damping M is a combination of \hat{G} and \hat{G} terms. When the centre of mass is fixed as it is in a wind tunnel, $\hat{G}=\hat{G}$, $\hat{Z}=0$. Hence la becomes

$$J \frac{d^2\theta}{dt^2} + (D_{M_p} - M) \frac{d\theta}{dt} + (K_{M_p} - M_{\theta})\theta = P(t)$$

In the Laplace domain, neglecting initial conditions,

$$\Theta^{(s)}\{Js^2 + (D_{M_p} - M)s + (K_{M_p} - M_{\Theta})\} = P(s)$$

$$\frac{P(s)}{\Theta(s)} = Js^2 + (D_{M_p} - M) \le + (K_{M_p} - M_{\Theta})$$

If P(t) is white noise (or an impulse), $P(\omega)$ is rlat. Hence equation. 2 can be written more simply in the frequency plane as

$$\frac{constant}{\Theta(s)} = J\omega_{p}^{2} + (D_{M_{p}}-M)j\omega_{p} + (K_{M_{p}}-M_{\Theta})$$

The bandwidth at half power point of the corresponding power spectrum can be shown to be (3)

$$\Delta \omega_{\rm p} \simeq 2 \omega_{\rm p} \zeta$$

where $\omega_{\mathbf{p}}$ = frequency in pitch. Using this we have

$$\Delta\omega_{\mathbf{p}} = \frac{(D_{\mathbf{M}_{\mathbf{p}}^{\cdot}} - \mathbf{M})}{\mathbf{J}}$$

M=0 with the wind off and we can measure $\frac{D_{M_P}}{J}$. J is gotten either from the geometry of the model and its mass or using a step input and measuring the natural frequency 'see 4 for details).

$$M = J(\Delta \omega_{Poff} - \Delta \omega_{Pon})$$

Hence
$$\dot{C}_{M_{\alpha}} + C_{M_{\dot{\alpha}}} = \frac{M}{\frac{1}{2}\rho u_{\infty}^2 SD}$$

Now for pure plunging motion in the z direction we have

$$m \frac{d^2z}{dt^2} + D_{M_L} \frac{dz}{dt} + K_{M_L} z = L(t) + L_A(t) \qquad .6$$

Again using the same argument as above, if L(t) is white noise and $L_A(t) = L_\alpha \alpha + L_\alpha \dot{\alpha}$, with α = angle of attack = $\tan^{-1} (\frac{\dot{z}}{u_\infty})$.

Nose up to the wind $\stackrel{\triangle}{=}$ positive angle of attack dictating the sign to be +.

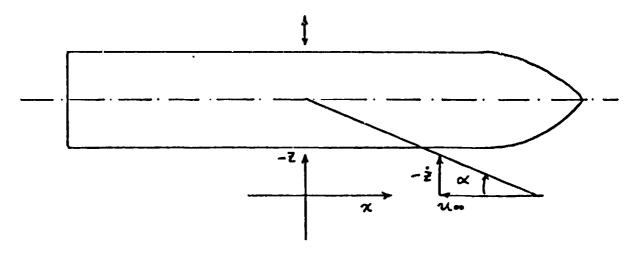


Figure 2 Origin of lift from a.

^{*}See Etkin. Dynamics of Atmospheric Flight. Wiley, 1972.

There is also a lift force due to a. Now for small

$$\frac{\dot{z}}{u_{\infty}}$$
, $\tan^{-1}(\frac{\dot{z}}{u_{\infty}}) = \frac{\dot{z}}{u_{\infty}}$

and
$$\dot{\alpha} \triangleq \frac{\dot{z}}{u_{\infty}}$$

Hence, equation 6 becomes

$$m \frac{d^2z}{dt^2} + D_{M_L} \frac{dz}{dt} + K_{M_L} z = L(t) + L_{\alpha} (\frac{\dot{z}}{u_{\infty}}) + L_{\dot{\alpha}} (\frac{\ddot{z}}{u_{\infty}})$$
 9

and again if L(t) is white, then we can experimentally determine

$$(m - \frac{L_{\alpha}}{u_{\infty}})$$
 and $(D_{M_{T}} - \frac{L_{\alpha}}{u_{\infty}})$

by measuring the bandwidth at 1/2 power points as

$$\Delta\omega_{L} = \frac{\left(D_{M_{L}} - \frac{L_{\alpha}}{u_{\alpha}}\right)}{\left(m - \frac{L_{\dot{\alpha}}}{u_{\alpha}}\right)}$$

and
$$\omega_{n_L}^2 = \frac{K_{M_L}}{(m - \frac{L_{\dot{\alpha}}}{u_{\infty}})}$$

or

$$\Delta\omega_{L} = \frac{(D_{M_{L}} - \frac{L_{\alpha}}{u_{\infty}})}{(R_{M_{L}})} \omega_{n_{L}}^{2}$$
12

Having gotten L_{α} and L_{α} we can relate them to

$$C_{L_{\alpha}} = \frac{L_{\alpha}}{\frac{1}{2} \mu u_{\infty}^{2} S}; \quad C_{L_{\alpha}^{\bullet}} = \frac{L_{\alpha}^{\bullet}}{\frac{1}{2} \rho u_{\infty}^{2} S}$$

Hence, $C_{L_{\alpha}}$, $C_{L_{\dot{\alpha}}}$, $(C_{M_{\dot{\alpha}}} + C_{M_{\dot{\theta}}})$ are determined.

In principle C_M and C_L can be determined from a plunging oscillation by measuring the currents and forces directly. However, two effects make this more difficult than using natural resonances. First, because of the large inertial loads, forces must be determined from differences in large numbers which have both phase and amplitude. Second, a small offset between the center of mass and the center of magnetization produces a moment as a result of vertical acceleration which must also be subtracted out. Further experience is needed to determine if these methods can be used

effectively and if these techniques can be used in the side slip planes.

The first step in incorporating Digital control of the magnetic balance system was to provide force and position inputs to the computer and position set signals to the balance. Figure 3 shows the block diagram for this interface.

COMPUTER INTERFACE

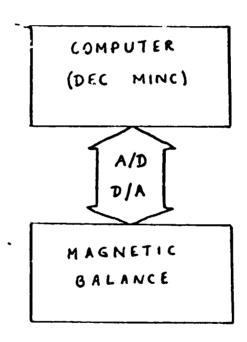


Figure 3 Computer Balance Interface.

A Digital Equipment Corporation minicomputer, the MINC 11/23, was used for the estimation of the stability derivatives. The E.P.S. signals from the balance provide six positions which, together with the six magnet currents, were sampled at 385 Hz simultaneously and recorded on an 8-inch floppy disk.

Two Data Translation analog-to-digital boards were used—a 16 channel single ended board that was used 'pseudo differentially' (the common side of the instrumentation amp was referenced to the magnetic balance ground) and an eight channel differential input board.

The signal levels being very low (typically $< \pm 1V$) the boards were set up to operate with a full-scale range of 1V. The 14 bit boards hence had a resolution of $\frac{1.V}{8192}$ or approximately 0.1 mV.

The cables were shielded, twisted pair and there did not seem to be any significant degradation of the signals as seen at the computer.

An extensive library of support programs was written for

- 1. Input/output of data
- 2. Display of data
- 3. Plots of data, and
- 4. Processing of data.

Appendix A lists some of the programs that were felt to be useful in the understanding of the processing technique. Most of the actual processing programs are interactive and require very little effort on the part of the experimenter. A concise procedure for the estimation is as follows:

I With the model hanging at the desired position, the Program 'Getdta' - gets in the positions and currents and saves them on a user specified file.

This process is repeated for different wind speeds. At each operating point four data files are recorded:

- a. A wind-off noise off data
- b. A wind-on noise off data
- c. A wind-on noise on data
- d. A wind-off noise on data

'noise' is the white noise source that is input to the desired degree of freedom.

- II The Program 'Zerome' (Zero means) is next run to take out the dc characteristics, and the operating point dc offsets for each file.
- III The Program 'DISPLA' displays any given data file and plots a graph of any given parameter. It also has the capability of doing a Fourier transform of the data and displaying the FFT.

This program also plots the graph on a line printer if desired. This is basically a program to evaluate the data and make quick 'by sight' decisions as to the validity of the results.

IV Program 'FILTER' filters out the dynamics of the magnetic balance.

V The program 'Model' finds the performance function of the system using a least squares error criteria. This is again an interactive program that uses a guess of the model order. The residual error energy is monitored and the model order chosen that has a reasonably small residual energy.

'Model' also performs a Fourier transform of the performance function which can be plotted.

The half power bandwidth of this resulting spectrum is measured and the appropriate parameters calculated. To calculate the initial balance parameters and the moment of inertia:

The program 'IGTDTA' - (Impulse and get data) may be used. This pulses the model with a small (1°) impulse and records the resulting damped oscillations. Using methods described in (3), the J and the natural frequency are evaluated.

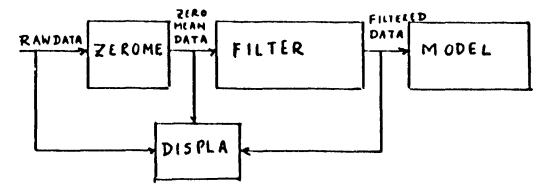


Figure 4 Size of blocks correspond approximately to computation time.

Wind tunnel tests and results including error analysis

Wind Tunnel Tests - The model was aligned with the tunnel axis in yaw and slip and lift and centered in drag. It was then set up at a 2-degree pitch angle.

White noise at 2 intensities was used as an input

+ 0.125V maximum and + 1V maximum which corresponded approximately to about 0.05 degrees and 0.2 degrees. Later, repeat
runs were made at 0.05, 0.1, 0.2 and 0.4 degrees. Data was
taken for four conditions:

- 1. Wind off noise off
- 2. Wind off noise on
- 3. Wind on noise on, and
- Wind on noise off.

These are shown in Figures 5 - 8.

Except in one region, the spectral power did not change when the wind or the white noise was turned on, as can be seen from the figures. The magnetic suspension feedback system poles are distinctly visible with the one varying peak corresponding to the model.

Since the amplitudes of the various other peaks are comparable to the one due to the model, the analytical model formed will not be a very accurate measure of the airplane model. The data was first filtered using a digital filter of length 64 using the Ramez exchange program (2).

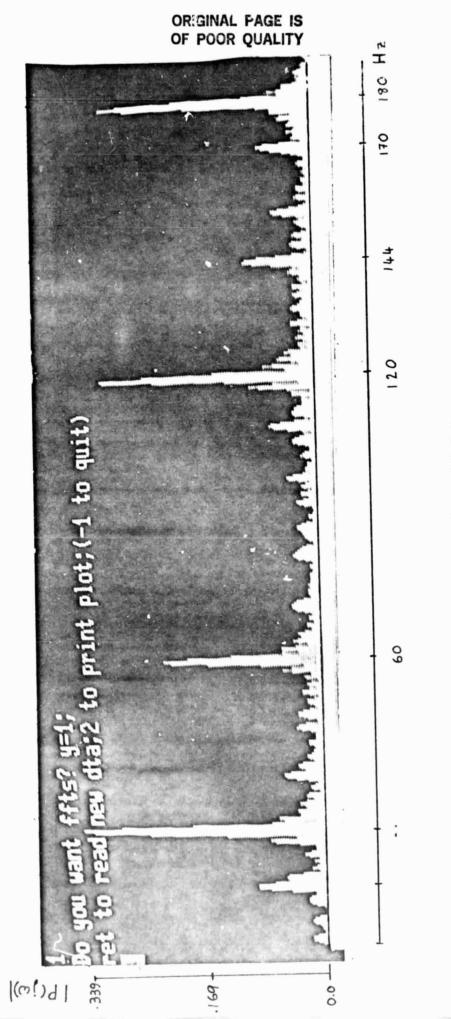


Figure 5 FFT of wind off noise off data.

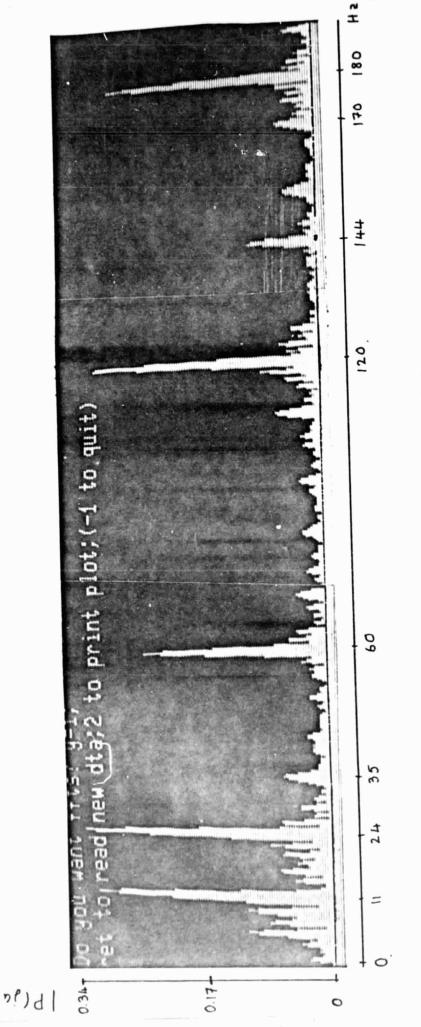
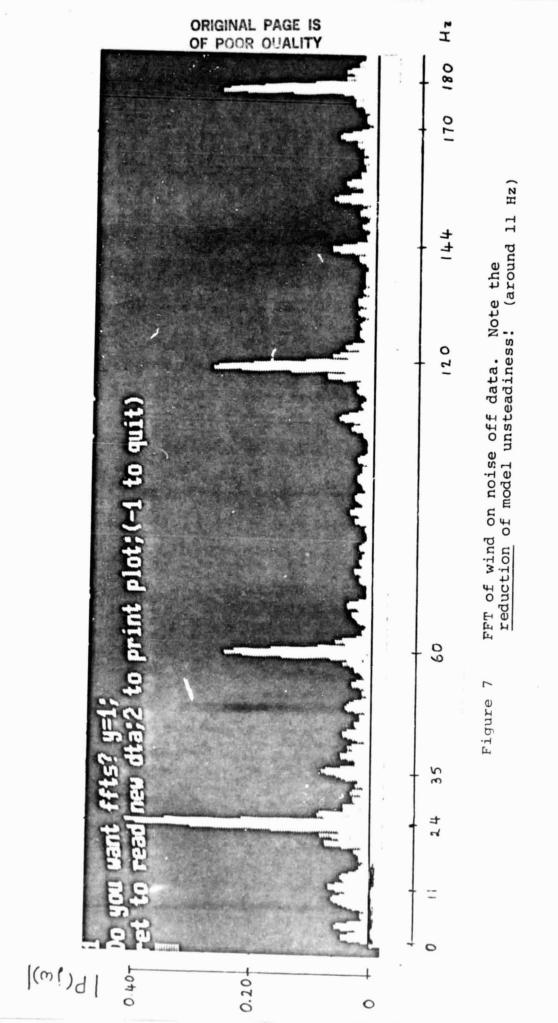
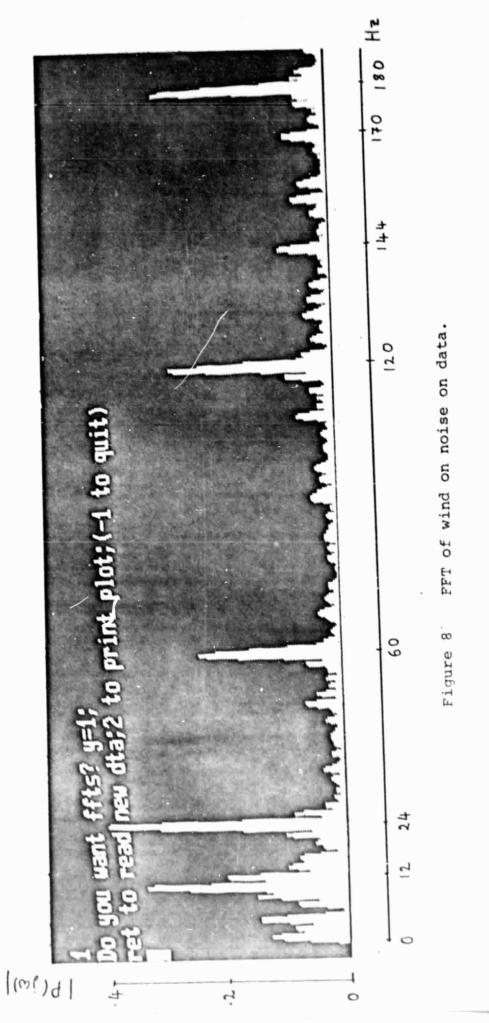


Figure 6 FFT of wind off noise on data.





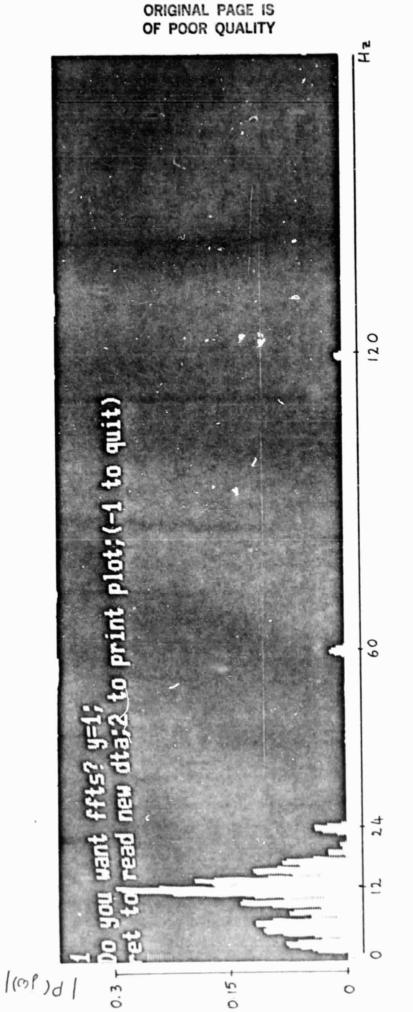
A pass band was used extending to 18 Hz and a stop band extending from 22 Hz. The pass band ripple was designed to about 0.65 db and stop band ripple was -20 db.

The choice of this filter was made arbitrarily. If better accuracy in the parameters is desired, the length can be increased at the cost of computation time. The effect of the filter was dramatic, as can be seen from Figures 8 and 9.

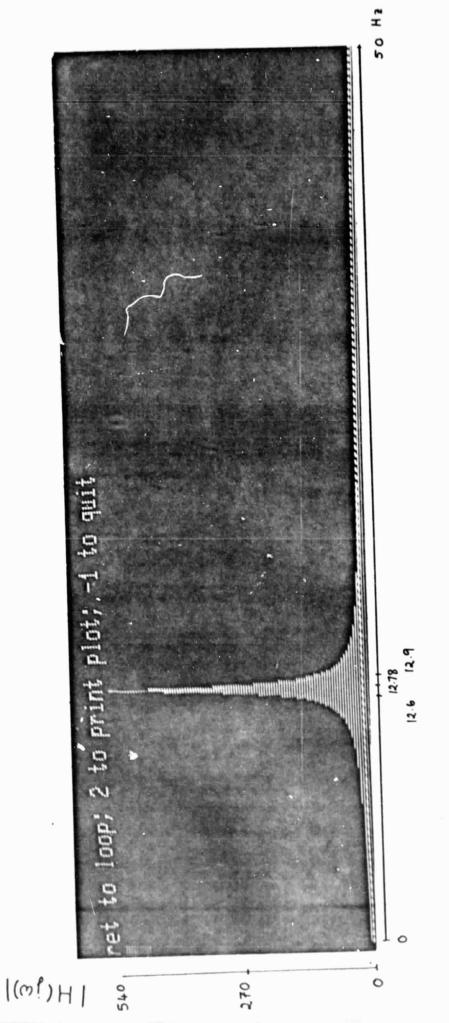
It is worth pointing out that the remaining points (past 22 Hz) cannot be just chopped off because of the ringing that results between samples (Gibbs phenomenon).

It is also worth pointing out that the modified signal is still only a small signal length (original sample time) and hence, still governed by the 'uncertainty principle'.

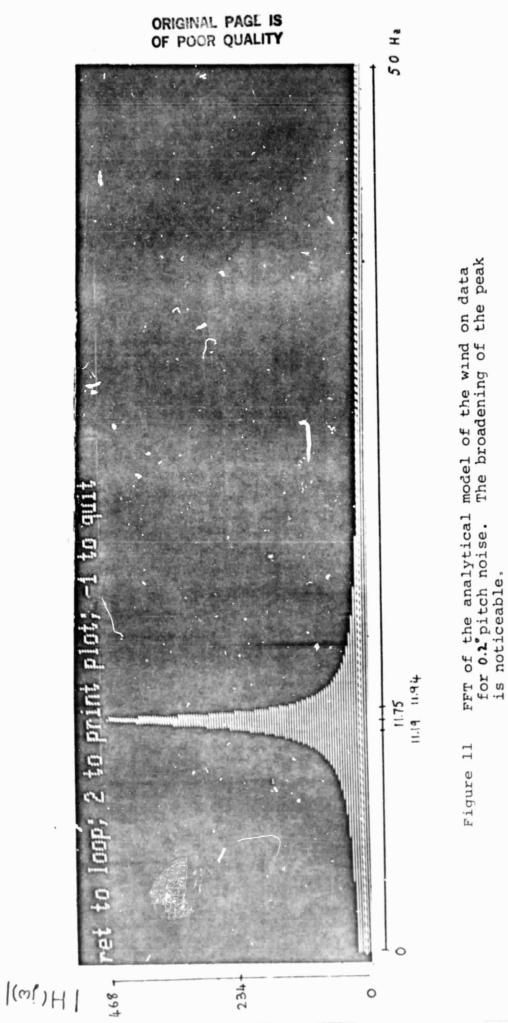
The signal was modelled using an autocovariance method and the stationarity of the residual energy when model order five was reached taken to be a sign of good fit. The Fourier transform of the model position is shown in Figures 10 and 11 for the wind off and on respectively. The excellent resolution in frequency is noteworthy.



The little FFT of wind-on noise on data after filtering. The lipeaks at 60 and 120 are due to the non-ideal nature of the filter. Figure 9



FFT of analytical model of the wind off data for 0.2 pitch noise. Figure 10



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Figure 11

Sample Calculations

The model used had the following properties:

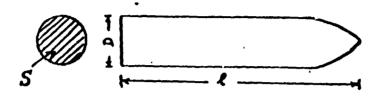


Figure 12

$$l = 12.5 \text{ cms.} = 0.125 \text{m}$$

$$\rho = 1.19 \frac{\text{kg}}{\text{meter}^3}$$
 at 0 altitude, $T = 20^{\circ}\text{C}$

m = 345 gms.

$$\frac{1}{2} \rho u_{\infty}^2 = 4.92 \times 10^3 \text{ Newtons/meter}^2$$

$$\frac{1}{2} \rho u_{\infty}^2 SD = 0.06$$
 Newton meters

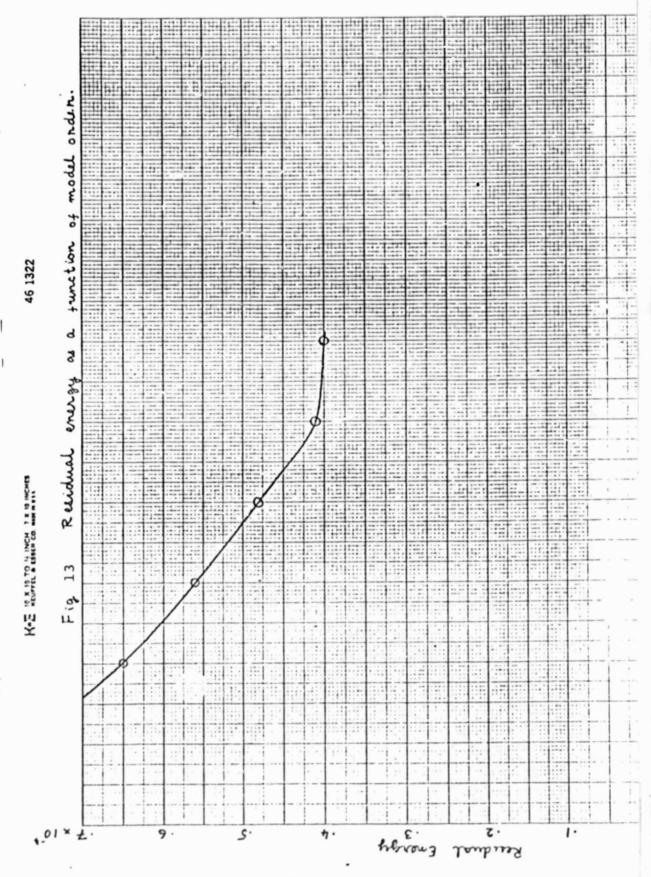
$$\frac{1}{2} \rho u_{\infty}^2 S = 2.41$$
 Newtons

$$u_{\infty} = 300 \text{ ft/sec} = 90.9 \text{m/sec}$$

D = 0.0254 meters (1 inch)

Figure 14 is the white noise, as seen on a scope and Figure 15 is a pitch position response, as seen on the computer.

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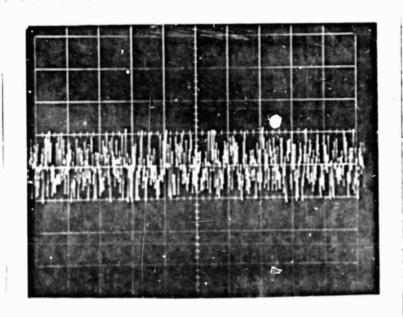


Figure 14

Typical white noise input.

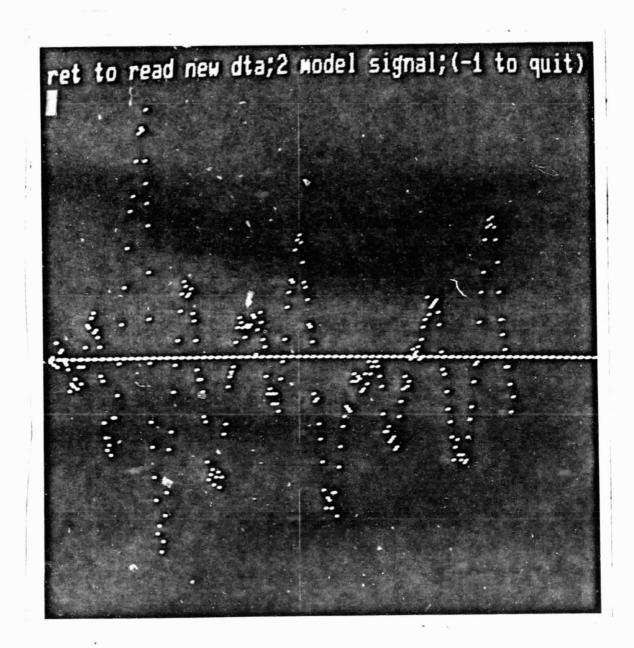


Figure 15

Typical model response in pitch.

Results

Results of the measurements are summarized below in Table 1. Runs A and B were made first. The extensive repairs were made on the power supplies, the controls were readjusted, and a week later Runs C, D, E and F were made. From examining Table 1 it is apparent that resolution is good but scatter from Run to Run is large, particularly with respect to the natural frequency.

Preliminary Pitch Damping Data for Ogive Cylinder Model at 300 ft/sec, $\alpha=2^{\circ}$, $Re_{D}=0.5\times10^{6}$

Table 1

Run	(a) Noise Amplitude Degrees	wn radi	Δ ω lans	Damping Ratio	(C _M , + C _M)* Dimensionless	U ft/se
A	.05	86.8	20.7	0.11	•	0
A	.05	79.7	14.2	0.09	+ 154.6	300
В	.2	80.3	1.8	0.011	-	0
В	.2	73.8	4.7	0.032	- 71.6	300
	Repeat	ed 1 wee	k later	after balan	ce repair and tunin	g
С	. 4	87.4	12.4	0.071	-	0
C	. 4	76.2	18.3	0.12	- 143.1	300
D	. 2	93.9	12.4	0.066	-	-
D	, 2	92.1	20	0.109	- 186.1	300
E	.1	111	42.5	0.19	•	-
E	.1	86.8	47.2	0.272	- 114.5	300
F	. 05	119 .8	46.1	0.192	•	-
F	.05	115.1	40.2	0.175	+ 150.3	300

This is a conventional notation with $C_{M_{\overset{\circ}{\alpha}}} = \frac{\partial C_{M}}{\partial (\frac{\partial \overset{\circ}{\alpha}}{2U_{-}})}$ and $C_{M_{\overset{\circ}{q}}} = \frac{\partial C_{M}}{\partial (\frac{\partial \overset{\circ}{\theta}}{2U_{-}})}$.

It is obtained from the values of Reference 4 by dividing by $\frac{d}{2U_m}$.

This indicates that the control system should have been adjusted to produce the same wind off natural frequency for each data point. Also, a larger data sequence should be tried to reduce scatter.

The negative damping (positive coefficient) observed at .05° amplitude is probably a result of system noise. Since noise tends to broaden the response curve, it is surmised that at very low excitation the decrease in noise resulting from damping on the wind offsets the broadening caused by aerodynamic damping. At higher excitations where the only power points are well above the noise level, the effect is not important. This suggests that excitation should be .2 to .4 degrees.

An estimate of the relative error can be made as follows:

$$e = \sqrt{\left(\frac{\delta \rho}{\rho}\right)^2 + \left(2 \frac{\delta u}{u}\right)^2 + \left(2 \frac{\delta D}{D}\right)^2 + \left(\frac{\delta t}{t}\right)^2} + \left(\frac{\delta m}{M}\right)^2 + \left(\frac{\delta \Delta \omega}{\Delta \omega}\right)^2 + \left(\frac{\delta A/D}{A/D}\right)^2$$

$$= \sqrt{\left(\frac{0.005}{1.19}\right)^2 + \left(\frac{2\times0.5}{300}\right)^2 + \left(\frac{2\times0.05}{2.5}\right)^2 + \left(\frac{0.05}{12.5}\right)^2} + \left(\frac{0.05}{12.5}\right)^2 + \left(\frac{0.5}{345}\right)^2 + \left(\frac{0.005}{18}\right)^2 + \left(\frac{0.1}{200}\right)^2$$

e = 0.041; i.e., - 4% error.

These are typical values.

This holds for repeated experiments at the same amplitudes.

The scatter in the three experimental values of runs D.E.F (dynamic stability derivatives -143,-186,114) may be related to the shift in stiffness in runs. The change in value with amplitude may be due to an additional error made in the fit due to the S/N being different for the three cases but further experiments should clarify these issues.

As with any new technique of magnetic balance experimentation, a sizeable base of experience is needed to establish the best way to obtain a given type of data.

Summary and conclusions

A technique using random noise excitation for estimating dynamic stability derivatives using the magnetic suspension system has been implemented. The method is not sensitive to drift and noise in the system. It also has the advantage of being simple and computationally not demanding. A 64K Byte MINC was adequate. Typical turnaround times from the start of the experiment to the evaluation of dynamic stability derivatives is of the order of an hour.

Theoretical calculations based on linear theory and assuming an $x_{ref} = 2.75$ inches from the nose, give a value of -286 for $C_{M_{c}} + C_{M_{c}}$ for this body.

Theoretical calculation of $C_{M_{\Omega}}$ gives a value of 1.477. The experimental values obtained by measuring natural frequency are 3.32, 0.9 and 5.94 corresponding to 0.1°, 0.2° and 0.4° noise.

Since $C_{M_{\alpha}}$ is computed directly from the change in stiffness wind on, the scatter is probably due to the change in wind off stiffness from run to run. If this can be eliminated, data scatter should be greatly reduced.

More work is needed to determine the cause of this scatter and whether static derivative estimation using this method can be competitive to the method of measuring currents and inferring the forces and moments acting on the body.

Some interesting facts about the balance can be seen from Figures 5 to 9 with the wind on, the balance is very quiet. Compare this power (Figure 5) to the spectrum with the wind on (Figure 7 This says that tunnel unsteadiness alone is not enough to excite the frequencies in the model for reasonable resolution.

It is reassuring to note that the system model derived by the white noise method $(s^2+12.44s+87.6^2)$ is very close to that obtained by Peter Luh using forced oscillation techniques $(s^2+12s+85.21^2)$ (5).

Two hundred and fifty-six data samples were taken at 385 Hz, which corresponds to approximately 1 second of data. This sampling rate for twelve channels is more than proportionately lower than the 15 KHz possible for only one channel.

It may be possible to increase this to nearer 1 KHz with soft-ware changes. However, increasing the sample rate well above the natural frequencies of the signal has adverse effects. It tends to cluster the poles right on the unit circle and the resolution of poles close together is reduced. 385 Hz is more than adequate since the highest frequency in the system seems to be around 30 Hz.

The simultaneous acquisition of data from all channels has the added advantage of being very useful for the evaluation of cross couplings between different degrees of freedom.

Essentially the same procedure may be used to estimate the effect of an input in one degree of freedom on another.

Static stability derivatives may be measured simultaneously to the dynamic ones by computation of the mean value of the signal with the wind on compared to the mean value with the wind off. However, this method has not been compared with direct force measurement for accuracy.

P

The parameter estimation programs can be incorporated as a subroutine in another program that analytically computes the half power bandwidth from the system performance function and hence, directly outputs the dynamic stability derivatives. However, a direct involvement in the mechanics of the procedure was retained in order to make sure that the numbers made sense.

The 64K Byte memory limitation on the MINC restricted taking more than 256 samples at a time. An additional 64K which is on order will eliminate this limitation. Ideally,

the number of samples should be approximately 100 times model order for a good estimate of the performance function.

The length 64 FIR filter has a stop band ripple of -20 dB and pass band ripple of 0.65 dB. This was arbitrarily chosen and the accuracy of the derivatives may be improved by increasing the length of the filter. This will increase computation time, however.

An important thing to keep in mind is that different model dynamics require a change in the filter characteristics (see Appendix D).

More experiments need to be done with models whose characteristics are either known or analytically determined and the results evaluated. Once this is done the system is ready for use as an easy to use and extremely versatile balance that can perform experiments until now not possible.

Appendix A

REFERENCES

Listed in the order referenced:

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Appendix B

This is the channel table for the magnetic wind tunnel computer interface.

C	.	cla	Singl	e End	ded		V	affay cla	1	fferent	 Liel I	nput
- 0		1	Bras	Posi	lion')	-1	Li	ft Curi	rent'	
1		2	Slip	Posi	tion	;	l	2	Ya	w Curre	ent	
2		3	Yau F	osit	ion	1	?	3	Pi	tch Cu	rrent	
3		.4	Lift	Posi	tion	;	5	4	Dr	as Curi	rent	
. 4		5	Pitch	Pos	ition		.	5	Si	de fore	e Cut	rent
							5	4	Ha	dnetiz	ing Cu	rrent
 -	!		·		<u>.</u>		<u></u>	7	"In	rut Sis	inal -	•
										•		
-											:	
D.	i_	iles and	their	inte	rpret	ation	· ·					
u	NDFFn	.DAT	Raw d	ists :	uith	wind	off	noise				
n 	NDFFn NDFNn	. DAT . DAT	Raw d	ista i	uith uith	wind wind	off off	noise	on			
n 	NDFFn NDFNn NDNFn	.DAT .DAT .DAT	Raw d Raw d	ists i	uith uith uith	wind wind wind	off off	noise noise	on off			
n 	NDFFn NDFNn	.DAT .DAT .DAT	Raw d Raw d	ists i	uith uith uith	wind wind wind	off off	noise	on off			
- 'U - 'U - 'U	NDFFn NDFNn NDNFn NDNNn	.DAT .DAT .DAT .DAT	Raw d Raw d Raw d	iata (with with with with	wind wind wind wind	off off on on	noise noise noise	on off on			
- W - W - W - W	NDFFn NDFNn NDNFn	.DAT .DAT .DAT .DAT	Raw d Raw d Raw d Raw d	iata i iata i iata i	with with with with with	wind wind wind wind	off off on on on	noise noise	on off on data			
- W - W - W - W - W	NDFFn NDFNn NDNFn NDNNn	.DAT .DAT .DAT .DAT .DAT	Raw of Raw of Raw of Zero	iata i iata i iata i iata i	with with with with with wind	wind wind wind wind	off off on on nois	noise noise noise e off e on	on off on data data			
- u - u - u - u - u - u - u - u	NDFFn NDFNn NDNFn NDNNn NDNNn WDFFn	.DAT .DAT .DAT .DAT .DAT .DAT	Raw of Raw of Raw of Zero	iata i iata i iata i iata i mean mean	with with with with wind wind wind	wind wind wind wind off	off off on	noise noise noise e off	on off on data data data	· ·		
- W - W - W - Z - Z - Z	NDFFn NDFNn NDNFn NDNNn NDNNn NDFFn NDFNn	.DAT .DAT .DAT .DAT .DAT .DAT .DAT	Raw of Raw of Raw of Zero Zero Zero	iata i iata i iata i iata i mean mean mean	with with with wind wind wind	wind wind wind wind off off on	off off on on nois nois nois	noise noise noise e off e on e off e on	on off on data data data data	• • • •		
- u - u - u - u - z - z - z - z	NDFFN NDFNN NDNFN NDNNN NDFFN NDFNN NDFNN NDFNN	.DAT .DAT .DAT .DAT .DAT .DAT .DAT .DAT	Raw of Ra	ista i ista i ista i ista i mean mean mean	with with with wind wind wind wind wind	wind wind wind wind off off on on	off off on	noise noise noise e off e on e off e on	on off on data data data data			
- W - W - W - W - W - W - W - W - W - W	NDFFN NDFN NDNFN NDNN NDFFN WDFFN WDFN WD	DAT DAT DAT DAT DAT DAT DAT DAT DAT	Raw of Raw of Raw of Zero Zero Zero Zero Filte	lata i lata i lata i lata i mean mean mean	uith with with wind wind wind wind wind	wind wind wind wind off off on on	off off on	noise noise noise e off e on e off e on	on off on data data data data data	· · · · · · · · · · · · · · · · · · ·		
- W W W Z Z Z Z Z Z Z F F F F	NDFFN NDFN NDNFN NDNN NDNN NDFN NDFN ND	DAT DAT DAT DAT DAT DAT DAT DAT DAT	Raw of Raw of Raw of Zero Zero Zero Filte	lata i lata i lata i lata i mean mean mean mean	with with with wind wind wind wind wind	wind wind wind off off on off	off off on	noise noise noise e off e on e off e on	on off on data data data data data			

Gautham Ramohalli M.I.T., Cambridge 8-may-81

```
Sun 03-May-81 15:38:29
```

```
C THIS PROGRAM GETS IN THE 6 CURRENTS, 5 POSITIONS AND THE INPUT
CFRON THE HAGNETIC BALANCE AND SAVES THEN ON DISK ON A FILE
CTHAT YOU ASK IT TO
                                  Gautham Ramohalli '
                                  M.I.T. Cambridge
C
      PROGRAM GETDTA
      LOGICAL*1 LIN, BLANK
      DIMENSION LIN(10). BLANK(10).
      INTEGER I.J.K
      REAL X(256,5),Y(256,7),SUM, MEAN
      INTEGER IX(254)
      N=256
      DATA X.Y/1280*0.,1792*0./
      WRITE(5,500)
500
      FORMAT(' ENTER 10 BLANKS')
      READ(5,501; BLANK
501
      FORMAT (10A1)
      DO 555 I=1,10
1
       LIN(I)=RLANK(I)
555 .
      CONTINUE
      CALL A2DH(0,0,5,N,X,1.)
      CALL A2DM(1,0,7,N,Y,1.)
      CALL SCAL(Y(1,7),IX,N)
      SUM=0.
      DO 200 I=1.N
       SUM=SUM+Y(I,7)
200
      CONTINUE
      HEAN-SUM/FLOAT(N)
      CALL CLEAR
      CALL GRAPH(N.IX)
      WRITE(5,999) HEAN
      FORMAT(' Hean Value=':F8.5;' The data displayed is the white noise
999
     X input')
      WRITE(5,998)
      FORMAT(' Do you want this data written to a file? w=1, Read wo
998
     Xre=0, auit=-2')
      READ(5.997) KR
      1F(KR.EQ.-2) GO 70 1000
      IF(KR.NE.1) GO TO 1
      CALL CLEAR
      WRITE(5,996) .
997
      FORHAT(12)
      FORMAT(' Enter the name of the file to be saved as')
996
      READ(5,357) LIN
      FORMAT (10A1)
557
      OPEN(UNIT=1.NAME=LIN.FORM='UNFORMATTED'.TYPE='NEW')
      DO 1200 I=1.N
       WRITE(1) (X(I,J),J=1,5) .
       URITE(1) (Y(I,J),J=1,7)
1200
      CONTINUE
      CLOSE (UNIT=1)
      60 TO 2
1000 CALL CLEAR
```

0

```
This program takes an input file and filters a given channel
C and writes the file back as a filtered tile. The program uses
  a 64 length finite impulse response filter with a rass band
  that extends to 18Hz and stop band that starts at 27Hz. This
C effectively filters out the magnetic balance system feedback
C effects from the signal and what remains is a pure signal
C due to the airplane dynamics!
CCC
                                   Gautham Ramohalli
                                   8-may-81
                                   M.I.T., Cambridge
      Program filter
      dimension LIN(10), LOUT(10), BLANK(10)
      LOGICAL#1 LIN, LOUT, BLANK, F. b
      Real x(256,5), y(256,7), h(64), rdup(7).
      complex xx(1024), hh(1024)
      Integer i.j.k.ichnl.flag
     data b/1HP/
      OPEN(UNIT=1, NAME='FILTER. DAT' : TYPE='OLD')"
       READ(1,*)(h(i),i=1,32)
    | close(unit=1)
      do 8081 i=1.32
       h(i+32)=h(33-1)
8081
      continue
      do 8082 i=1.64
      hh(1)=h(1)
      write(6,8083) i,h(i)
8083
      format(' h(',12,') = ',E20.9)
      continue
8082
      do 8084 1=45,1024
      hh(1)=0.
8084
      continue
      call foures(hh.1024,-1)
      WRITE(5,950)
950
      FORMAT(' ENTER 10 BLANKS PLEASE')
      READ(5.501) BLANK
501
      FORMAT (10A1)
1
      DO 1000 i=1,10
       LIN(i)=BLANK(i)
       LOUT(i)=BLANK(i)
      continue
1000
      write(5,999)
999
      format(' enter the name of the input file')
      read(5,998) LIN
998
      format(10A1)
      urite(5,958) LIN
      urite(5,2716)
      format(' enter the channel to be filtereil(es :5) ')
      read(5,2717)r,1chnl
      format(A1, 12)
      write(5,2718)r,ichn1
      format(' filter channel '.A1.12.'?')
```

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```
VI NE
                                                             PAGE 002 .
           V02.5
                      Fri 08-May-81 05:34:07
      read(5,2719) kr
2719
      format(12)
      if(kr.ea.-1) so to 1
758
      format(' file name=',10A1)
1040
      continue
      flas=0
      if(r.ne.b) flas-1
      OPEN( UNIT=1, NAME=LIN, FORM='UNFORMATTED', TYPE='OLD')
      do 1100 i=1,256
      read(1) (x(1.j).j=1.5)
      read(1) (y(1,j),j=1,7)
1100
      continue
      close(unit=1)
      if(flag.ea.1)so to 1200
      do 1150 1=1,254
       xx(i)=x(i\cdot ichn1)
1150
      continue
      60 TO 1250
1200 do 1250 i=1,256
       xx(i)=v(i,ichn1)
1250
      continue
      do 1270 i=257.1024
       xx(i)=0.0
1270
      continue
      call foures(xx,1024,-1)
      do 1280 i=1,1024
       xx(i)=xx(i)*hh(i)
1280
      continue
      call foures(xx,1024,1)
      if(flas.ea.1) so to 1310
      do 1290 1=1,254
       x(i,ich-l)-Resl(xx(i))
1290
      continue
      so to 1320
1310
      do 1320 1=1,256
       v(i,ichn1)=Real(xx(i))
1320
      continue
      format(/,' enter the name of the output file')
997
957
      FORMAT(' input file='.10A1.' output='.10A1)
      urite(5,997)
      read(5,998) LOUT
      write(5,957) LIN, LOUT
      OPEN(UNIT-2, NAME=LOUT, FORM='UNFORMATTED', TYPE='NEW')
      do 1500 1=1,256
         urite(2) (x(i,J),J=1,5) '
         urite(2) (u(i,j),j=1,7)
1500
      continue
      close (unit-2)
      urite(5,990)
990
      format(/,' Bo you have any more files to work on? yol')
      rebd(5,787) kr
787
      forest(12)
      if(kr.eq.1) so to 1
      stor
```

(

99A

775

774

urite(5,975)

ur:te(5,994)

write(5,793)

```
This program reads in a input file ( magnetic balance data)
  and performs a covariance method of signal modelling
  The program is interactive and asks for the file name
  and the length of data and model order. It loops back
 for a change in model order.
                                   Bautham Ramohalli
C
                                   3-may-81
C
                                   M.I.T..Cambdridde
C
      Program Model
      logical#1 LIN.blank.r.b
      dimension LIN(10).blank(10)
      double precision X(256),A(21), KC(21),yy(300),A2(21)
      double precision ALPHA
      real rdum(7), w(512), xx(512), PI
      integer no.m.mp.i.J.k.IX(512).ff.ichnl
      complex z(1024),W
      DATA b: w/1HP+51240./
      11-12
      PI=ATAK(1.)#4.
      W=CMPLX(Q.,PI)
      write(5,15) b
15
      format(1x+A1)
      NO 17 1-1,256
       X(1)=0.
17
      continue
      do 17 1-1-21
       RC(1)=0.
       A(1)=0.
17
      continue
      ALPHA-O.
      nn=256
      urite(5,500)
500
      format(' Enter a blank line please')
      read(5,501) blank
501
      format(10A1)
999
      format(' Enter the name of the file to be read')
      do 5 1=1,10
1
      LIN(i)=blank(i)
3
      continue
      urite(5,999)
      read(5,998) LIN
778
      format(10A1)
      urite(5.997) LIN
997
      format('1 File name is '.10A1)
      urite(5,994)
```

format(/, P1=drag position', T40, 'C1=lift current')

format(' P2=slip rosition'. T40. 'C2=yau current')

format(' F3=you position'. T40. 'C3=pitch current')

format('1To display a particular parameter type the appr no')

```
PAGE 002
            V02.5
                      Fri 98-Haw-81 07:41:49
       urite(5,992)
       format(' P4=lift position', T40, 'C4=drag current') .
 992
        urite(5,991)
. 991
        format(' P5=pitch position',T40,'C5=slip current')
        urite(5,990)
 990
        format(T40, 'C6=magnetizing current')"
        urite(5,989)
 989
        forwat(T40,'C7=input sidnal')
        read(5,988) r,ichnl
 988
        format(A1, I2)
        w*ite(5,987) r.ichnl
       "o mat('1',' display ',A1,I2,'data?,(-1 to read file asain)')
 987
        ...d(5.986) kr
        if(kr.ca.-1) so to 1
       call clear
        if(r.ea.b) so to 1190
        OPEN(UNIT=1, NAME=LIN, FORM='UNFORMATTED', TYPE='OLD')
        do 1100 i=1,nn
           read(1) (rdum(j), j=1,5)
           read(1) (rdum(J),J=1,7):
           y(i)=rdum(ichnl)
           X(i)=rdum(ichn1)
 1100
         continue
        close(unit=1)
         so to 1200
        OPEN(UNIT=1, NAME=LIN, FORM='UNFORMATTED', TYPE='OLD')
 1170
        'do 1250 i=1,nn
           read(1) (rdum(j);j=1,5)
           v(i)=rdum(ichn1)
           X(i)=rdum(ichn1)
           read(1) (rdum(j), j=1,7)
 1250
        continue
            close(unit=1)
 1200
        continue
            call scal(v,IX,256)
        call graph(256,IX)
 1275
        urite(5,985)
        format(' ret to read new dta;2 model signal;(-1 to quit)')
 985
        read(5,98%) kr
 984
        format(I2)
        call clear
        if(kr.ea.-1) so to 3000 '
        if(kr.eq.2) so to 2500
       so to 1
2500
       urite(5,969)
       format(' Make sure the line printer is set up right!')
. 969
       format(' The vertical lri=6, formlensth=66')
 96B
        read(5,967) kr
 967
       format(12)
 557
       urite(6,777) ff
       write(6,970) LIN.r.ichnl
       format(///* Datafile='+10A1+T30+'Pos/Cur='+A1+T50+'Chrl='+12
 970
 777
       format(A1)
```

Ţì.

```
urite(6,979)
979
      FORMAT(/, -----
                       -----/,//)
      urite(5,950)
950
       format(' model order=?')
      read(5,951) a
       format(I2)
951
      write(6,952) m
952
       format(' model order=',13)
      urite(5,953)
       format(' length of signal=?')
953
      read(5,954) n
      format(14)
      call covar(n:X:m:A:ALPHA:RC)
      ar=a+1
      write(6,955) n
      format(/, ' length of the signal=', 14)
     -urite(6,956)
956
      format(/,T22,'model coefficients',T50,'raflection coefficient
      do 350 i=1.mp
     " write(6,957) (i,A(i),RC(i),i=1,mp)"
       format(/,13,17X,E20,10,10X,E20,10)
957
350
      continue
       write(6,958) ALPHA
      format(/, ' Residual energy=', E20.10)
958
      if(m.1t.7) so to 8080
      -urite(6,777) ff
8080
      urite(6,979)
      idua=2*a+256
      n=idum
      do 937 i=1.m
       yy(i)=0.
       yy(idum-i+1)=0.
937
      continue
      do 959 i=1,256
       vy(i+a)=X(1)
959
      continue
      call auto(n, yy, m, A2, ALPHA, RC)
      urite(67955) n
      write(6,956) .
      do 800 i=1.mp
       write(6,957) (i,A2(i),RC(i),i=1,mp)
800
      continue
      write(6,958) ALPHA
      URITE(6:979)
      urite(5,555)
      format(' ret to change model orderil to read new file;
555
     X 2 to set ffts of models;-1 to auiti')
      read(5,986) kr
      if(kr.ne.2) so to 8092
       urite(5,7070)
7070
      format(' which model do you want ffts for covar=0:auto=1')
       read(5,986) kr
       flas=0.
```

44

stop

Appendix C

LIST OF SYMBOLS.

D _{ML}	-22	Magnetic damping in lift
K _M L		Magnetic stiffness in lift
D _M P		Magnetic damping in pitch
K _{MP}		Magnetic stiffness in pitch
ζ		Damping ratio
ω		Frequency in radians/sec
J		Moment of inertia about the center of gravity
u_		Free stream velocity in the -x direction
q		9D/2u_
O		Pitch angle with respect to the x axis
M		Moment acting on the body
a		Angle of attack
ρ		Density of air
m		Mass of body
s		Characteristic area
		Characteristic length
≜		By definition is

L = aerodynamic lift force on the body.

$$C_{\underline{L}} = \frac{\underline{L}}{\frac{1}{2} \rho u_{\underline{\omega}} S}$$

$$C^{\mathbf{L}^{\alpha}} = \frac{9\alpha}{9 C^{\mathbf{L}}}$$

$$C_{L_{\dot{\alpha}}} = \frac{\partial C_{L}}{\partial \dot{\alpha}}$$

$$C_{M} = \frac{M}{\frac{1}{2} \rho u_{\infty}^{2} SD}$$

$$C_{M\dot{\Theta}} = \frac{\partial C_{M}}{\partial \dot{\Theta}}$$

$$C_{M_{\dot{\alpha}}} = \frac{\partial \dot{\alpha}}{\partial \dot{\alpha}}$$

NOTE. THIS DEFENITION LEADS TO UNITS OF SECS. FOR

Appendix D

FILTER DESIGN GUIDE

The Remez exchange algorithm needs the following inputs:

Card 1 N. ITYPE, NBANDS, NGRID

N - order of filter. (This is an interactive procedure where you keep increasing N until the desired attenuation in the stop band and pass band ripple is achieved.)

N=1: Low pass high pass or band pass

NBANDS = Number of bands, eg., a low pass

filter has two bands.

NGRID: Grid density always set to sixteen. For example, a length fifteen low pass filter would have the following first card: 15, 1, 2, 16

Card 2 Fllow, Flhigh, F2LOW, F2HiGH

FNBANDS LOW, FNBANDS HIGH

This card-specifies the edges of the bands. There are NBANDS as specified on Card 1. These band edges are entered as real numbers (not more than four per line) and normalized to (0.0, 0.05) interval corresponding to the interval (0-1/2T) where T is the sampling time. In this case 1/2T = 187 Hz.

Thus a low pass filter with pass band to 18 Hz, stop band from 22 Hz would have Card 2. 0.0, 0.0467, 0.057, 0.5

Card 3 AMP1... AMPNBANDS

This card specified the desired magnitude response in each band. For example, a low pass filter would have

1.0, 0.0

Card 4 Wl. ... WNBANDS

Card 3

Specifies the ripple weighting in each band. These are real numbers. The ripple ratio is given by δ_1/δ_i , where δ_1 is the maximum derivation allowed in the first band and δ_i is the derivation allowed in the ith band.

Thus for a low pass filter with $\delta_1 = 0.1$, $\delta_2 = 0.2$, this card would be 1.0, 0.5

The result of running this program is a data file called FILTER.DAT that gives the impulse response. This file should then be transferred to the disk where the FILTER.SAV program resides (i.e., the same disk that you have your magnetic balance data). The FILTER.SAV program reads in the filter characteristics before it operates on the magnetic balance data file.

Detailed instructions for the use of this procedure will be found in a user manual that is being planned and should be available at the balance.

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*****************
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Gautham Ramohalli,

W91/106, (3-8334)

M.I.T., Cambridge, Na. 02139

.....INPUT FILE.....

16 64 1 2 0.057100002 0.50000000 0.00000000 0.046799999 0.00000000 1.00000000 2.00000000 1.00000000 CALLING REHEZ AND RUNNING PROGRAM. DONT INTERRUPT DEVIATION = -0.000231585DEVIATION = -0.039413721**DEVIATION = 0.132467500** DEVIATION = 0.150773944 DEVIATION = -0.155212869DEVIATION = -0.156569904DEVIATION = -0.156635265DEVIATION = -0.156635449

FINITE IMPULSE RESPONSE (FIR)
LINEAR PHASE DIGITAL FILTER DESIGN
REMEZ EXCHANGE ALGORITHM

BANDPASS FILTER

FILTER LENGTH = 64

**** IMPULSE RESPONSE **** H(1) = -0.76954342E-01 = H(64)H(2) = 0.33773184E-02 = H(63)H(3) = 0.45766905E-02 = H(62)H(4) = 0.63917860E-02 = H(61)H(5) = 0.85576177E-02 = H(60)H(6) = 0.10764614E-01 = H(59)H(7) = 0.12635142E-01 = H(58)H(8) = 0.13839498E-01 = H(57)H(9) = 0.14070064E-01 = H(56) $^{\circ}$ H(10) = 0.13136059E-01 = H(55) H(11) = 0.10920897E-01 = H(54)H(12) = 0.74827075E-02 = H(53)H(13) = 0.29894039E-02 = H(52)H(14) = -0.21979585E-02 = H(51)H(15) = -0.76648444E-02 = H(50)H(16) = -0.12855738E-01 = H(49)H(17) = -0.17256759E-01 = H(48)H(18) = -0.19906990E-01 = H(47)H(19) = -0.20732537E-01 = H(46)H(20) = -0.19146100E-01 = H(45)H(21) = -0.14848895E-01 = H(44)H(22) = -0.77870712E-02 = H(43)

```
H(23) =
                     0.19024462E-02 = H(42)
            H(24) =
                     0.13866879E-01 = H(41)
            H(25) =
                     0.27566157E-01 = H(40)
            H(26)
                     0.42291783E-01 = H(39)
            H(27) =
                     0.57208348E-01 = H(38)
            H(28) =
                     0.71434379E-01 = H( 37)
            H(29) =
                     0.84089853E-01 = H(36)
            H(30) =
                     0.94392702E-01 = H(35)
            H(31)
                     0.10165238E+00 = H( 34)
            H(32) =
                     0.10535892E+00 = H(33)
                       BAND
                                     BAND
 LOWER BAND EDGE
                     0.000000
                                   0.0571000
 UPPER BAND EDGE
                     0.0468000
                                   0.5000000
 DESIRED VALUE
                     1.0000000
                                   0.000000
 WEIGHTING
                     2.0000000
                                   1.000000
 DEVIATION
                     0.0783177
                                   0.1566354
 DEVIATION IN DB
                     0.6549352
                                 -16.1021996
 EXTREMAL FREQUENCIES -- HAXINA OF THE ERROR CURVE
    0.0117187
                0.0273437
                            0.0400391
                                        0.0468000
                                                     0.0571000
    0.0649125
                                                     0.1264360
                0.0795609
                            0.0951859
                                        0.1108109
    0.1430375
                0.1586625
                            0.1742875
                                        0.1899125
                                                     0.2065141
    0.2221391
                0.2377641
                            0.2543657
                                        0.2699907
                                                     0.2856157
    0.3012407
                0.3178422
                            0.3334672
                                        0.3490922
                                                     0.3647172
    0.3813188
                0.3969438
                            0.4125688
                                        0.4281938
                                                     0.4447953
    0.4604203
                0.4760453
                            0.4916703
****************************
```